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# CARRYING CAPACITY OF NOTCHED COLUMNS

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**Abstract**—Studies were conducted on notched and unnotched columns subjected to axial compressive loading. The objective was to determine the effects of cracks of different depths and root radii on the maximum load carrying capacity of long and short columns with notches on one side and on opposite sides. Results indicate that the maximum load carrying capacity of eccentrically loaded columns having slenderness ratios less than 250 may be reduced significantly, particularly when these columns contain fatigue cracks or notches with a root radius smaller than 0·003 in.

## INTRODUCTION

THE stability of columns and other structural components has often been investigated [1–3]. Some effort has also been made to determine the weakening effects of flaws in specimens subjected to tensile, compressive or bending loads [4]. In most of these studies, however, the effect of flaws on the buckling or critical load was not treated [5, 6]. Consequently, the results presented in this paper are primarily concerned with obtaining the maximum load of long and short columns containing cracks. The loads were concentric and eccentric axial compressive loads, and the cracks were of varying depths and root radii on long slender and short columns having notches on one side and also on both sides.

The results presented in this paper may be of significance in many areas of engineering where slender columns are employed to achieve high-strength-to-weight performance. For example, the results are applicable to the aeronautical and aerospace industries, since aircraft and missiles have slender column construction in wing and fuselage sections. Flaws may develop in these structures from mechanical vibrations, aerodynamic loads, rocket fuel exhaust, leakage of rocket fuel or acoustical fatigue.

For these investigations $\frac{1}{2}\times\frac{1}{2}$ in. rolled bars made of 7075–T6511 aluminum were selected. The column specimens were made by cutting the bars to the desired lengths and facing off the ends. The columns containing cracks had notches put in at the center of the specimens, either on one side or on two opposite sides as required. On the adjacent sides that did not contain notches, small grooves were put in to minimize possible surface effects caused by the rolling operations of the manufacturing process.

Some columns were tested having machined notches of root radius of approximately 0·003 in. Other columns were fatigued at the notch to produce a very small crack by mounting one end in a fixture held eccentrically in the chuck of a 16-in. Hendley lathe and holding the other end rigidly by the tool post.

The depth of both the machined and the fatigued crack was measured after failure with an optical comparator. An r.m.s. value of the notch depth was calculated using ten depth readings at equal intervals for irregular cracks. For a crack of approximate uniform depth the average depth was found by use of the cross lines in the comparator.

## TEST EQUIPMENT

The columns were loaded in a fixture (Fig. 1) having knife edges at the ends with adjustments to enable concentric and eccentric loading. This assembly was mounted in a Tinius Olsen 120,000-lb-capacity universal testing machine. The movement of the loading head was recorded with a Tinius Olsen differential transformer. Dial gages, which were used to determine the amount of transverse displacement of the column, were mounted on a vertical stand. Vertical alignment was accomplished with dial gages to within 0·0005 in.

## TEST PROCEDURE

The load was applied to the columns by lowering the head of the testing machine at a constant rate. The applied load versus vertical displacement of the loading head was plotted automatically on a chart recorder. The transverse deflection of the column was also measured. The columns were loaded to failure or, in case of no failure, well beyond the point of maximum load.

## THEORETICAL CONSIDERATIONS

### *Critical load*

The Euler buckling load $P_{cr}$ for columns with pinned ends is well known [4] as

$$P_{cr} = \frac{\pi^2 Ebh^3}{12L^2} \tag{1}$$

where $E$ is Young's modulus, $b$ is the width of the column, $h$ is the depth of the column and $L$ is the length of the column. The length $L$ was taken as the length of the column plus the length of the knife edges. From Fig. 2 it is seen that the results of the theory and experimental findings agree quite well. Tests on various lengths of columns were performed; in particular, extensive tests were conducted on 12-in. columns.

### *Equations of stress*

It is possible to determine the approximate stress in columns subjected to an eccentric compressive load. Within the linear range this stress is equal to the sum of the compressive stress and the stress due to bending. The tensile stress due to bending could be used in the following equations, since the opening mode of the crack is of interest. Using beam column assumptions, the approximate stresses become

$$\sigma = \sigma_c + \sigma_b \tag{2a}$$

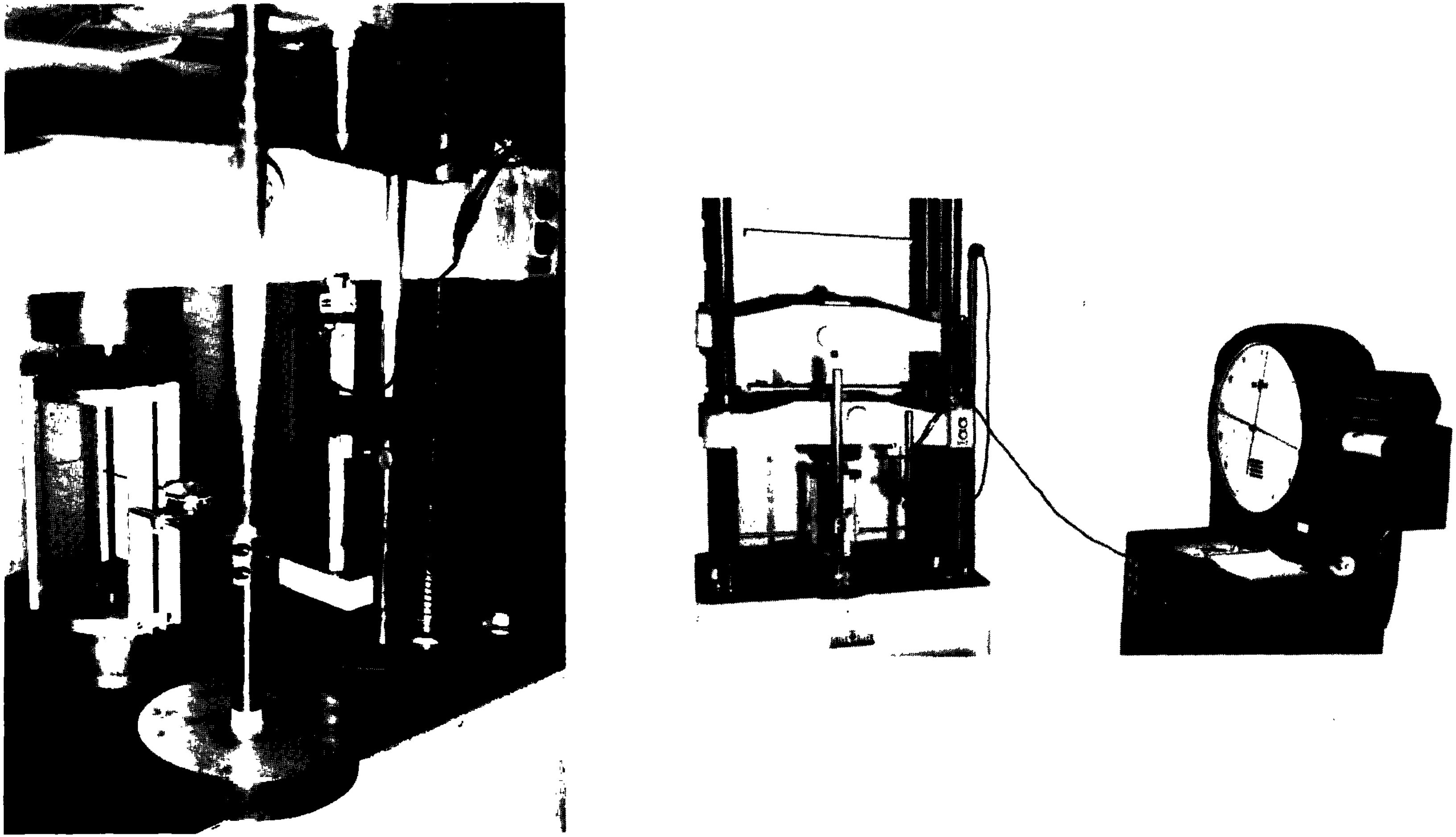


FIG. 1. Test equipment and specimen.

where

$$\sigma_c = \frac{-P}{bh} \tag{2b}$$

$$\sigma_b = \frac{12P(\epsilon+\delta)c}{bh^3} \tag{2c}$$

in which $P$ is the applied load, $c = h/2$ is the distance from the neutral axis to the outer fiber, $\epsilon$ is the eccentricity of loading, and $\delta$ is the horizontal deflection at the midpoint of the column. Finally, substituting (2b) and (2c) into (2a) results in

$$\sigma = \frac{P}{bh}\left[-1+\frac{6(\epsilon+\delta)}{h}\right]. \tag{3}$$

Equation (3) is valid for a uniform column without notches.

The approximate maximum stress could also be determined by employing stress concentration formulas developed by Neuber [4]. For example, he considered the effect of deep and shallow hyperbolic external notches for cases of pure tension and bending. The maximum stress at the root of the notch may be determined for tension as

$$\sigma_{\max} = \frac{P}{ab}\,\frac{[(a/\rho)+1]\sqrt{(a/\rho)}}{[(a/\rho)+1]\arctan\sqrt{(a/\rho)}+\sqrt{(a/\rho)}} \tag{4}$$

and for pure bending as

$$\sigma_{\max} = \frac{2P(\epsilon+\delta)(a/\rho)\sqrt{(a/\rho)}}{a^2b[\sqrt{(a/\rho)}+[(a/\rho)-1]\arctan\sqrt{(a/\rho)}]} \tag{5}$$

where $\rho$ is the radius of curvature of the root of the notch and $a$ is half the width of the net cross section at the root of the notch. Equations (4) and (5) may be combined with appropriate signs to give the resulting stress at a notch in a column subjected to an axial compressive and a bending load. The superposition of (4) and (5) is valid within the limits of the linear theory of elasticity.

One difficulty in using (4) and (5) is that the dimensions of the net section area are required to obtain the maximum stress for each incremental load. Obviously, errors are introduced in using (3), (4) and (5) for determining the maximum stress in notched columns. Furthermore, it is questionable whether Neuber's expression for stress concentration is valid for fatigued crack specimens where $\rho$ approaches zero. In view of the possible inaccuracies in applying the above and other available formulas to slender columns with notches, theoretical investigations are presently underway to extend the present stress concentration theory.

## RESULTS AND DISCUSSION

Tests were conducted on columns having lengths of 6, 9, 12, 15, 18, 24 and 30 in. The majority of the tests were conducted on columns 12 in. long, but an adequate number of tests were also performed on the other lengths.

The results for unnotched columns subjected to a concentric axial load are shown in Fig. 2. In these tests, with the eccentricity equal to zero, the columns were continually

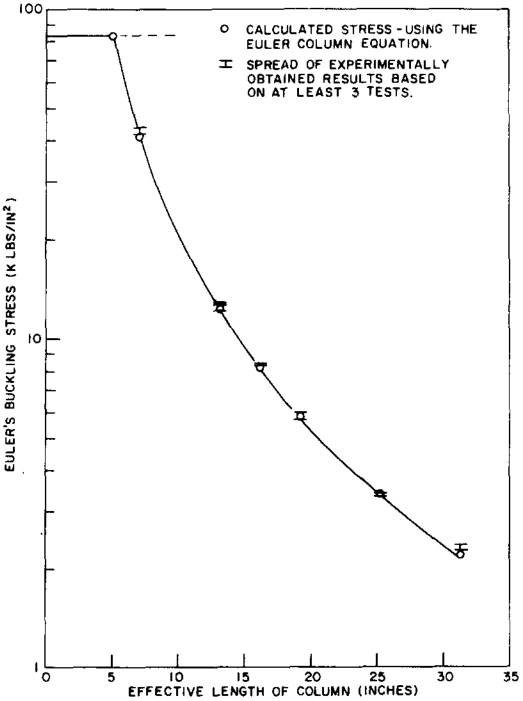


FIG. 2. Comparison of theoretically predicted Euler load with experimental results.

adjusted during the loading to insure straightness. The required adjustments were always very small. Some tests were also conducted using double notched columns. For the case of zero eccentricity, no appreciable difference for the Euler buckling load was observed for notched and unnotched columns. This result is apparently correct, since the column is subjected solely to an axial load and not permitted to bend. Consequently, the cracks close under compressive load and have virtually no effect on straight columns which are made to remain straight until buckling. A series of tests was also run on 12-in.-long unnotched columns at various eccentricities of loading (Table 1). In these tests the columns were loaded well beyond the inflection point on the load deflection curve.

TABLE 1. MAXIMUM LOADS OF THE 12-IN.-LONG UNNOTCHED COLUMNS HAVING DIFFERENT ECCENTRICITIES

| Eccentricity (in.) | Experimentally determined maximum load (lb) |
|---|---|
| 0 | 3045* |
| 0 | 3150 |
| 0·090 | 2440 |
| 0·250 | 2050 |
| 0·500 | 1590 |
| 1·000 | 1200 |
| 1·250 | 1020 |

* Euler buckling load which was calculated using a length of column plus the added length due to the knife edges.

The same eccentricities were used for the unnotched and notched columns. The depth of notch (or crack) was varied. The deflection in the horizontal direction was measured at approximately the center of the column. Load–deflection curves obtained for various crack depths are shown in Figs. 3–9. Although experiments were conducted on columns with one and two notches, Figs. 3–9 show only the results of columns with a single notch; the results for the double notch are discussed later. The notch was on the side of the column farthest from the axis of the applied load.

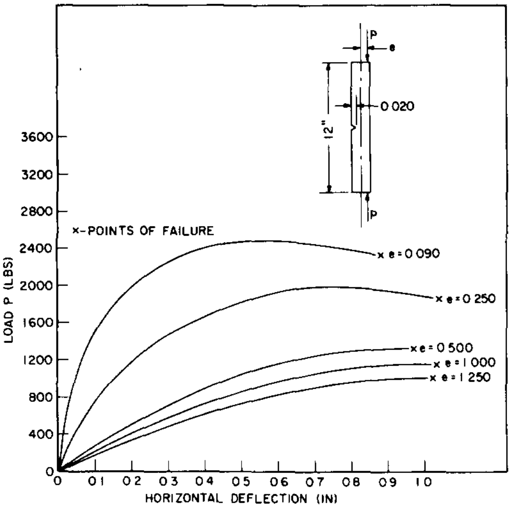


FIG. 3. Load–deflection curve for a column with a 0·020-in. deep notch with a fatigue crack extension.

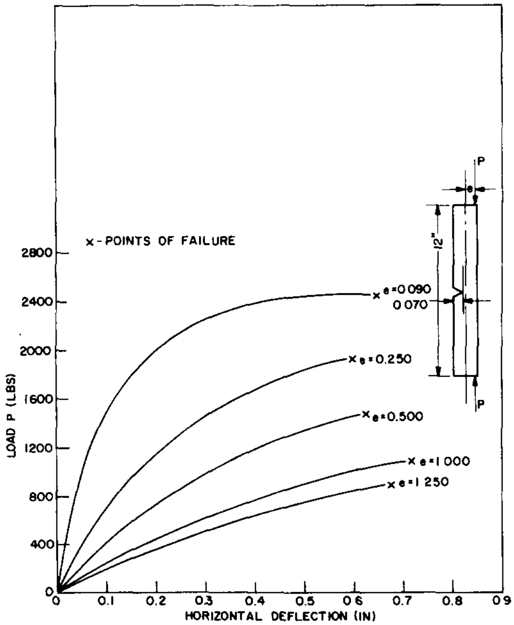


FIG. 4. Load–deflection curve for a column with a 0·070-in. deep notch with a fatigue crack extension.

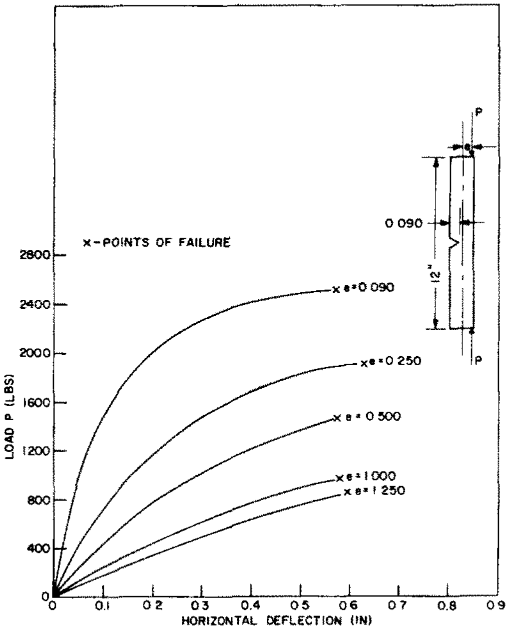


Fig. 5. Load–deflection curve for a column with a 0·090-in. deep notch with a fatigue crack extension.

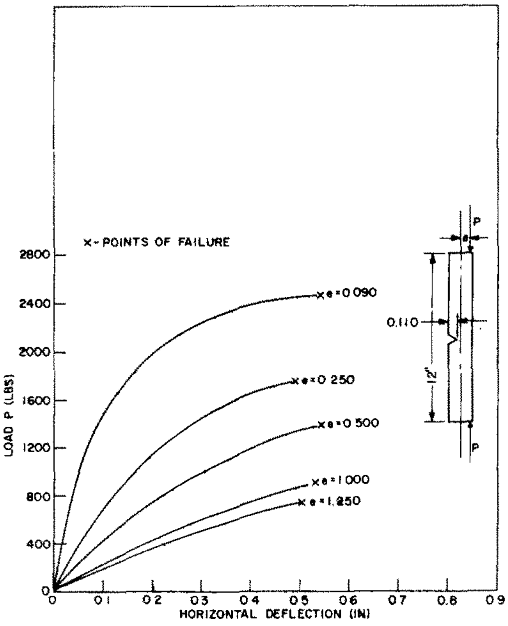


Fig. 6. Load–deflection curve for a column with a 0·110-in. deep notch with a fatigue crack extension.

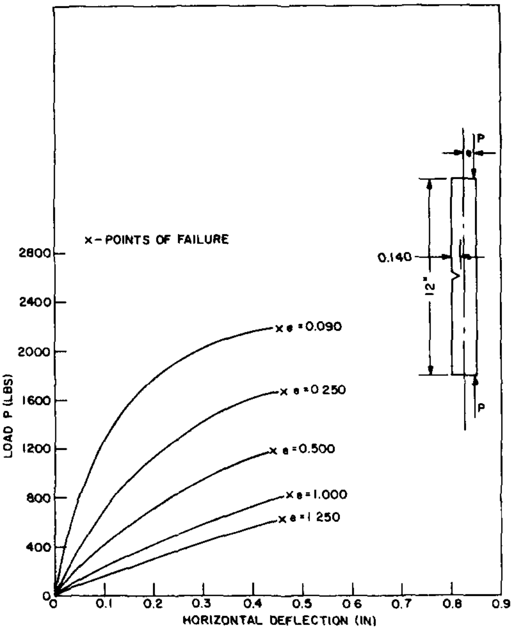


FIG. 7. Load–deflection curve for a column with a 0·140-in. deep notch with a fatigue crack extension.

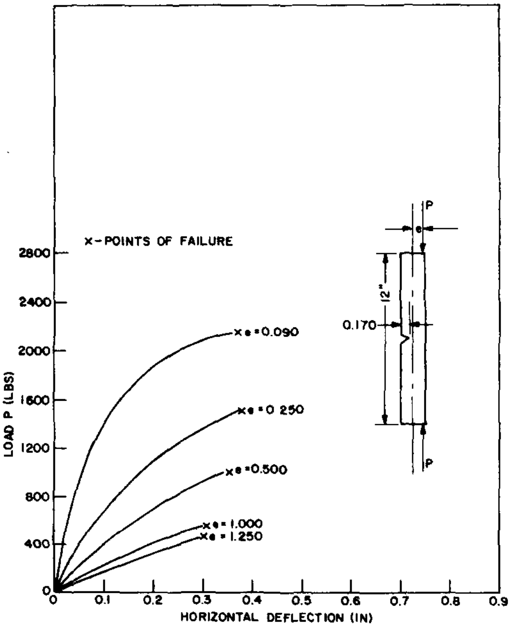


FIG. 8. Load–deflection curve for a column with a 0·170-in. deep notch with a fatigue crack extension.

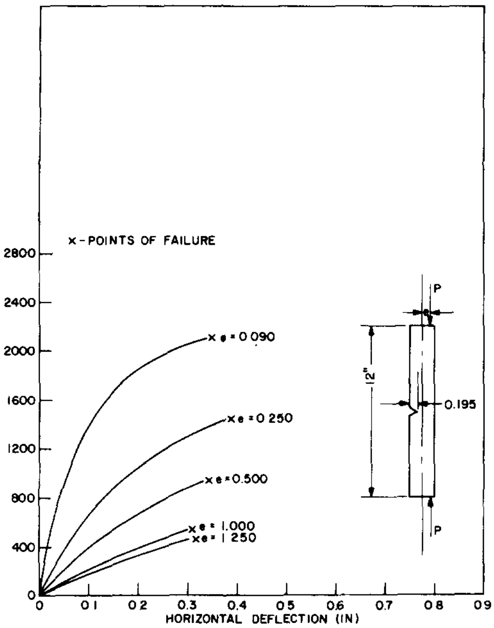


FIG. 9. Load–deflection curve for a column with a 0·195-in. deep notch with a fatigue crack extension.

The failure load was measured using seven crack depths at five eccentricities. A comparison between the response of the notched bars at various eccentricities with those of the unnotched bars is tabulated in Figs. 3–9. The comparison shows that a fatigue crack in a column under axial load reduces its load carrying ability. This load carrying capacity of the column is further reduced as the depth of the crack is increased. Furthermore, increasing the eccentricity of the applied load decreases the load carrying ability. Finally, it appears

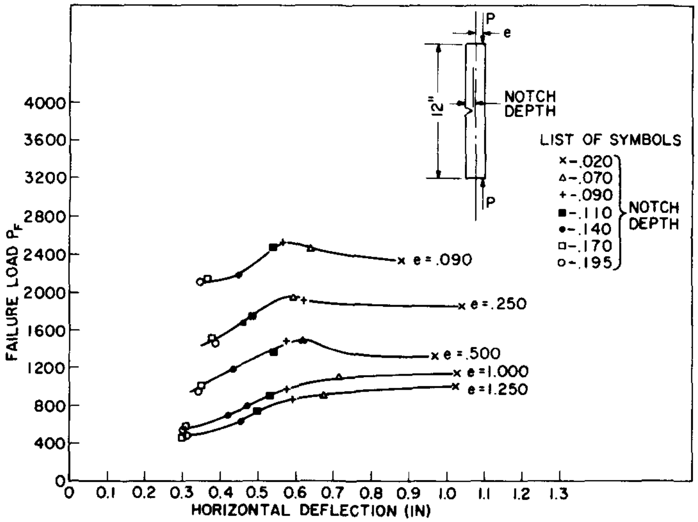


FIG. 10. Effect of notch depth on failure load and maximum horizontal deflection of the column. (All notches have a fatigue crack extension.)

that the horizontal deflection is approximately constant for a given crack depth. This can be seen from Fig. 10, where failure load has been plotted versus the corresponding maximum horizontal deflection. The largest decrease in load carrying ability from the notched to the unnotched column was 55 per cent, which occurred for a notch depth of 0·195 in. and an eccentricity of 1·250 in. The failure loads were plotted versus the notch depths at various eccentricities (Fig. 11). The values presented as zero notch depth are the maximum loads obtained for the tests on the unnotched columns for the eccentricities indicated.

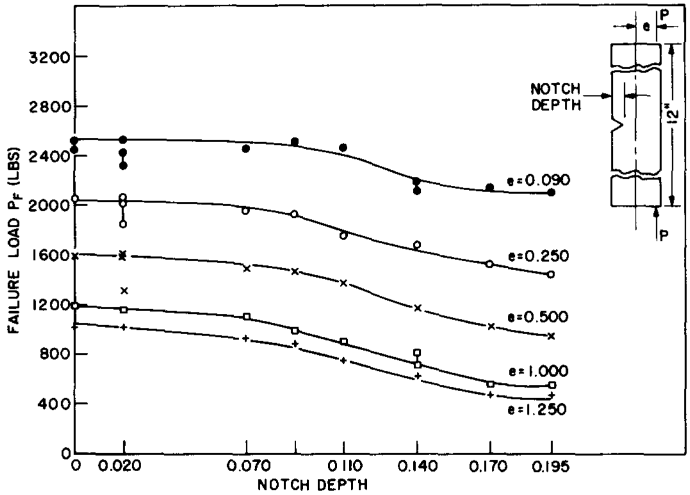


FIG. 11. Effect of depth of notch with a fatigue crack extension on failure load.

The effect of the root radius of the crack was also studied (Fig. 12). The majority of the results were obtained using a notch with a fatigue crack extension. In Fig. 12 results have also been given for 0·003-in. and 0·0625-in. root radii; the large root radius resulted in higher failure loads than those for small root radius for the tests conducted in this research.

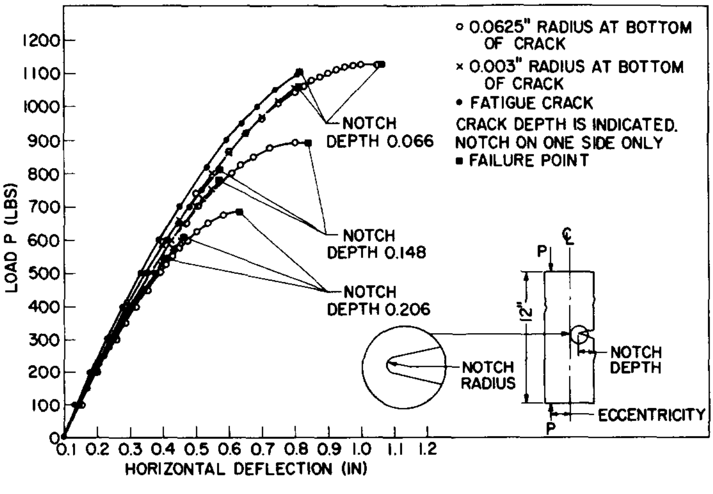
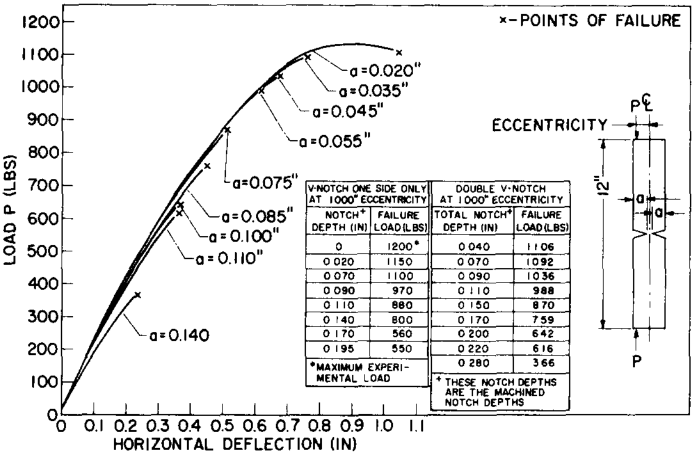


FIG. 12. Effect of notch depth and root radii on load carrying ability of columns. (Eccentricity = 1·0 in.)

No appreciable difference was found between the fatigue crack and the 0·003-in. radius crack. However, the 0·0625-in. root radius appears to have no effect on preventing the load from reaching a maximum value before failure, because the corresponding load deflection curves have approximately zero slope at failure.

The results of the effect of putting a notch on both sides of the column are shown in Fig. 13. Note that the total notch depth for the double V-notch indicated in the table on this figure is equal to the sum of the length of the crack on each side of the column. These tests indicate that a double notch is less disastrous than a single notch with a crack depth equal to the total crack depth of the double notch. Symmetry appears to be maintained for a double notch but is not maintained for a single notch.



| V-NOTCH ONE SIDE ONLY AT 1.000" ECCENTRICITY | |
|---|---|
| NOTCH⁺ DEPTH (IN) | FAILURE LOAD (LBS) |
| 0 | 1200* |
| 0.020 | 1150 |
| 0.070 | 1100 |
| 0.090 | 970 |
| 0.110 | 880 |
| 0.140 | 800 |
| 0.170 | 560 |
| 0.195 | 550 |

*MAXIMUM EXPERIMENTAL LOAD

| DOUBLE V-NOTCH AT 1.000" ECCENTRICITY | |
|---|---|
| TOTAL NOTCH⁺ DEPTH (IN) | FAILURE LOAD (LBS) |
| 0.040 | 1106 |
| 0.070 | 1092 |
| 0.090 | 1036 |
| 0.110 | 988 |
| 0.150 | 870 |
| 0.170 | 759 |
| 0.200 | 642 |
| 0.220 | 616 |
| 0.280 | 366 |

⁺ THESE NOTCH DEPTHS ARE THE MACHINED NOTCH DEPTHS

Fig. 13. Load–deflection for a column with two V-notches on opposite sides and of varying depth. (Eccentricity = 1·0 in.)

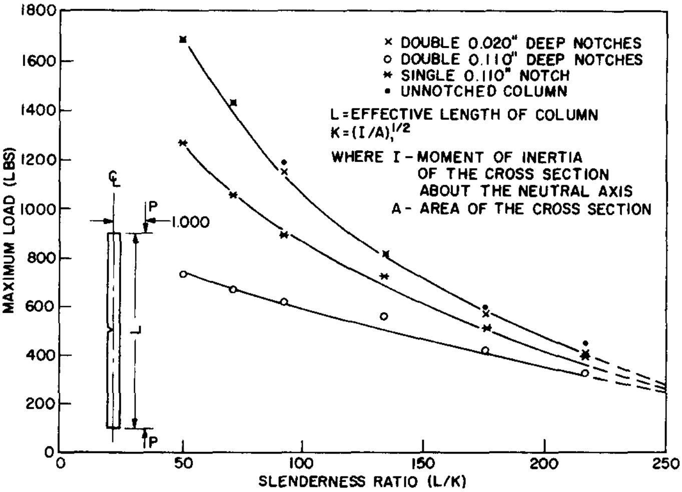


Fig. 14. Effect of notches and slenderness ratio on the maximum load carrying capacity of columns. (Eccentricity = 1·0 in.)

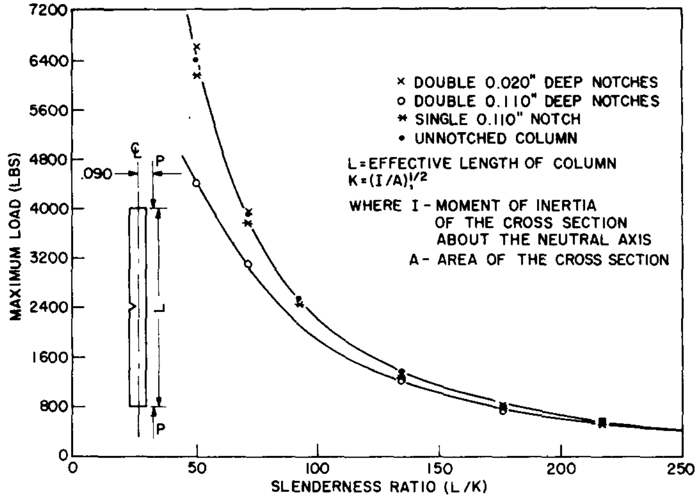


FIG. 15. Effect of notches and slenderness ratio on the maximum load carrying capacity of columns. (Eccentricity = 0·090 in.)

The results of a series of tests on the effects of slenderness ratio, depth of notch, and eccentricity on the maximum load carrying capacity of the column are shown in Figs. 14 and 15. Two eccentricities were considered, namely, 1·000 in. in Fig. 14 and 0·090 in. in Fig. 15. A variety of notches were considered, namely, a single 0·110-in.-deep notch and double notches 0·020 and 0·110-in. deep. From Figs. 14 and 15, it appears that the load carrying ability of 7075-T6511 aluminum columns having slenderness ratios larger than 250, and for eccentricities of 0·090 and 1·000-in., is not noticeably affected by notches. However, as the slenderness ratio decreases from a value of 250, the effect of notches on the load carrying capacity of the column increases. For small slenderness ratios the weakening effect of the notches is most pronounced, especially for the 1·000-in. eccentricity.

To determine whether a preferred orientation due to manufacturing processes existed in the rolled bar, four columns that were taken from the same bar were tested. A single identical notch was put into each of these columns but placed on their four different sides for the four tests. Differences in the load carrying capacity and transverse deflection were less than the experimental accuracy.

## CONCLUSIONS

For the aluminum 7075-T6511 columns considered, the following conclusions may be drawn:

1. Presence of a fatigue crack in a column reduces its load carrying capacity when the load is applied eccentrically in compression (Figs. 3–9).
2. The experimentally determined buckling load is not affected by the presence of two symmetrical and identical cracks for concentrically loaded straight columns.
3. For slenderness ratios less than 250, the sharp notches appear to have a significant effect on the load carrying ability of the columns (Figs. 14 and 15). This effect is especially pronounced for large eccentricity of load application. For slenderness ratios greater than 250, these notches do not appear to have quite as significant an effect on the load carrying ability.

4. The load carrying capacity decreases with increasing depth of notch for slenderness ratios less than 250 (Figs. 14 and 15).
5. For a column length of 12 in. the weakening effect of a fatigue crack becomes more pronounced when loads are applied at greater eccentricities (Figs. 3–9).
6. The reduction in load carrying ability of a 12-in. long fatigue cracked column having a slenderness ratio of about 92 is the same as for a sharp notch with a root radius of 0·003 in. or less, as long as all the other dimensions remain the same (Fig. 12).
7. A root radius of 0·0625 in. does not appear to result in catastrophic failures of the type exhibited by a fatigue crack in a column having a slenderness ratio of 92 (Fig. 12).
8. In loading columns to failure, a fatigue crack on one side of the column is more disastrous than a fatigue crack of half the depth on both sides of the column as may be seen in Fig. 15 for a slenderness ratio of 92.
9. For a slenderness ratio of 92 and a given depth of fatigue crack, failure appears to occur at a near constant horizontal deflection when the column is loaded eccentrically in compression (Fig. 10).
10. Columns made from rolled bar stock did not show any preferred orientation due to manufacturing processes in these studies.

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**Résumé**—Des études furent faites sur des colonnes entaillées et non entaillées soumises à des charges de compression axiales. Le but est d'établir les effets de fissures de profondeurs et de rayons d'origine différents sur la résistance à la charge maximum de colonnes longues et courtes avec des entailles sur un côté et sur les côtés opposés. Les résultats indiquent que la force de résistance à la charge maximum de colonnes à charge excentrique ayant des rapports de minceur inférieurs à 250 peuvent être réduits d'une façon considérable, en particulier lorsque ces colonnes contiennent des fissures ou des entailles de fatigue avec un rayon d'origine inférieur à 0,003 pouce.

**Zusammenfassung**—Studien wurden unternommen an gekerbten sowie ungekerbten Säulen, die axialen Drucklasten ausgesetzt wurden. Das Ziel war die Bestimmung der Einwirkung von Rissen verschiedener Tiefe und Wurzelhalbmesser auf die Tragkraft langer und kurzer Säulen mit Kerben an der Seite und an der gegenüberliegenden Seite. Die Resultate zeigen, dass die Maximaltragkraft excentrisch belasteter Säulen mit Schlankheitsverhältnis unter 250 bedeutend vermindert werden kann, insbesondere wenn diese Säulen Daueranrisse oder Kerben mit Wurzelhalbmessern unter 0,003 Zoll haben.

**Абстракт**—Рассматривается задачи о колоннах с вырезом или без выреза, нагруженных осевом давлением. Целью работы является определение эффектов щел, разной глубины или радиусов выреза, на максимальную нагрузку, исчерпывающию несущую способность в длинных и коротких колоннах, с вырезами с одной стороны или с обеих. Результаты утверждают, что максимальная несущая способность эксцентрично нагруженных колонн, обладающих отношениями гибкости менее чем 250, может быть значительно уменьшена, особенно когда эти колонны имеют усталостые щели или вырезы, которых радиусы менее чем 0·003 дюйма.